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NLEVP: A Collection of Nonlinear Eigenvalue Problems. Users' Guide.

Timo Betcke* Nicholas J. Higham† Volker Mehrmann‡

Gian Maria Negri Porzio† Christian Schröder‡ Françoise Tisseur†

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Abstract

This is the Users' Guide for NLEVP: a collection of nonlinear eigenvalue problems provided in the form of a MATLAB toolbox. A separate paper describes the collection and its organization.

1 Introduction

This document describes how to install and use the NLEVP MATLAB toolbox, which provides a collection of nonlinear eigenvalue problems.

For details of the design and organization of the collection, and the problems it contains, see [1].

This document describes 4.0 of the toolbox. The collection will grow and contributions are welcome (see Section 6).

2 Installation and Usage

To install the toolbox create the directory `nlevp` in a suitable location and make it the current directory. Then you can

- either clone the NLEVP toolbox if you are a Git user with the command

  ```
  git clone https://github.com/ftisseur/nlevp
  ```

  then add NLEVP to the MATLAB path (ideally in `startup.m`) with the command

  ```
  addpath(nlevphome), savepath
  ```

  or through the File menu;

- or download the toolbox it as a zip file from `https://github.com/ftisseur/nlevp`, then use appropriate "unzip" software (making sure to preserve the directory structure) Remember to put put the `nlevp` directory on the MATLAB path as above.

*Department of Mathematics, University College London, WC1E 6BT, UK (`t.betcke@ucl.ac.uk`). The work of this author was supported by Engineering and Physical Sciences Research Council Grant EP/H004009/1.

†School of Mathematics, The University of Manchester, Manchester, M13 9PL, UK ({`nick.higham,francoise.tisseur, gianmaria.negriporzio`}`@manchester.ac.uk`). This work of these authors was supported by Engineering and Physical Sciences Research Council grant EP/D079403/1. The work of the second author was also supported by a Royal Society-Wolfson Research Merit Award and Engineering and Physical Sciences Research Council grant EP/E050441/1 (CICADA: Centre for Interdisciplinary Computational and Dynamical Analysis) and the work of the fifth author was also supported by a Leverhulme Research Fellowship and Engineering and Physical Sciences Research Council grant EP/I005293/1.

‡Institut für Mathematik, MA 4-5, Technische Universität Berlin, Germany ({`mehrmann,schroed`}`@math.tu-berlin.de`). Supported by Deutsche Forschungsgemeinschaft through MATHEON, the DFG Research Center *Mathematics for key technologies* in Berlin.

To try the toolbox, run the demonstration script by typing `nlevp_example` at the MATLAB command prompt. Then execute the following commands:

```
help nlevp
nlevp query problems
nlevp query properties

nlevp help railtrack
nlevp query railtrack
coeffs = nlevp('railtrack')
spy(coeffs{2})

coeffs = nlevp('bicycle')
polyeig(coeffs{:})
```

The collection does not work with versions 7.6 (R2008a) and earlier versions of MATLAB, since it uses functionality introduced in MATLAB 7.7 (R2008b).

3 Release History

Versions 1.0–3.0 are available from

`http://www.mims.manchester.ac.uk/research/numerical-analysis/nlevp.html`

3.1 First Release, 1.0

The first release of the toolbox was version 1.0, dated 4-Apr-2008, and contained 26 problems.

3.2 Second Release, 2.0

The second release, version 2.0, was dated 15-Nov-2010, contained 46 problems, and had the following changes:

- Problem `string` has been renamed `spring`. `spring` has been generalized to include more parameters but is backward compatible with `string`. Invoking `nlevp('string')` still works: it invokes `nlevp('spring')` and produces a warning message.
- The matrices generated by problems `acoustic_wave_1d` and `acoustic_wave_2d` have been modified in order to more closely match the formulation in the paper from which this problem is taken. The eigenvalues now lie in the upper half-plane instead of the left half-plane.
- New problems are: `fiber`, `foundation`, `genhyper2`, `Hadeler`, `intersection`, `metal strip`, `pdde_stability`, `plasma_drift`, `omnicam1`, `omnicam2`, `qep1`, `qep2`, `qep3`, `qep4`, `railtrack2`, `relative_pose_5pt`, `relative_pose_6pt`, `shaft`, `speaker_box`, `surveillance`.
- New functionality: `nlevp('eval',...)` and `[coeffs,fun] = nlevp('name',...)`.
- Automatic testing of problem properties via `nlevp_test`.
- Cosmetic changes have been made to some of the functions.
- Citations to the sources of the problems have been updated, where necessary.

3.3 Third Release, 3.0

The third release, version 3.0, is dated 20-Dec-2011, contains 52 problems, and has the following changes:

- New problems `gen_tantipal2`, `gen_tpal2`, `mirror`, `planar_waveguide`, `qep5`, `time_delay`.

- For scalable problems the first dimension parameter now specifies the size, n, of the coefficient matrices (or an approximation to it). Previously, n was a function of this parameter in some cases. It is now possible to generate all scalable problems of a given size (or approximately that size). A warning message, with identifier `NLEVP:truescale`, is printed when the affected problems are called.

- Scalable problems now return coefficient matrices in the MATLAB `sparse` format when the coefficient matrices are sparse.

- A new problem property `random` has been introduced to specify problems that use random numbers in their construction. Such problems include an optional input argument that is used to seed the random number generator. If that optional input argument is not provided then the same (fixed) problem is generated each time. A random number seed argument has been added to `gen_hyper` and may result in different matrices being generated than with previous versions of NLEVP. All problems with the `random` property can use either the old or new (`rng`) MATLAB syntax for seeding the random number generator, as chosen through an input argument.

- `dirac` has been vectorized. The coefficients may differ at the level of rounding error from those produced by the previous, unvectorized code.

- The first two input arguments of `spring_dashpot` have been interchanged, so that the first is the dimension.

- A bug in the computation of the derivatives of the `gun` problem has been corrected.

- This release is compatible with GNU Octave [2], as far as possible. It has been tested with Octave 3.2.4 under Windows and Octave 3.4.3 under Linux. Since Octave does not have a function `polyeig` the function `nlevp_example` will not run.

3.4 Fourth Release, 4.0

The fourth release, version 4.0, dated 25-Mar-2019, contains 74 problems, and has the following changes:

- 22 problems have been added (see Table 1). Their description can be found in [3].

- A new third output is returned by all the problems. It is called `F` and returns the function handle for $F(\lambda) = \sum_{i=0}^{k} f_i(\lambda)A_i$. This new output should be used in place of

```
>> F = @(lam) nlevp('eval',name,lam)
```

  whose evaluation is slower for large size problems than that of the new third output;

- Three new identifiers are available, `tridiagonal`, `banded` and `low-rank`. The property `banded` is given to problems with coefficient matrices $A_i \in \mathbb{C}^{n\times n}$ having bandwidth less than $n/5$. The `low-rank` flag is used for problems that have at least one low-rank coefficient.

- For all the problems with the `low-rank` identifier, a fourth output, called `xcoeffs`, has been added. It returns a $2 \times (k + 1)$ cell array such that if the i-th coefficient matrix $A_i = E_iF_i$ for some rectangular matrices E_i and F_i, then E_i is equal to `xcoeffs{1,i}` and F_i to `xcoeffs{2,i}`.

- The syntax `randn('state',state)` to seed the random number generator has been replaced by the newer syntax `rng(state)`. The problems influenced by this choice are `bcc_traffic`, `gen_hyper2`, `gen_tantipal2`, `gen_tpal2`, `mirror` and `spring_dasphpot`.

- Minor bugs fixed.

- New GitHub repository for the MATLAB codes and documentation, `https://github.com/ftisseur/nlevp`.

- This release is not compatible with GNU Octave [2].

Table 1: New problems in NLEVP version 4.0.

Quadratic	bcc_traffic	circular_piston	damped_gyro
	deformed_consensus	disk_brake100	disk_brake4669
	elastic_deform	utrecht1331	
Rational	photonic_crystal	railtrack_rep	railtrack2_rep
Nonlinear	bent_beam	bucking_plate	canyon_particle
	distributed_delay1	nep1	nep2
	pdde_symmetric	pillbox_cavity	pillbox_small
	sandwich_beam	time_delay2	

4 The MATLAB Function nlevp

The toolbox has just one main user-callable function, `nlevp`, which is as follows.

```
function varargout = nlevp(name,varargin)
%NLEVP   Collection of nonlinear eigenvalue problems.
%  [COEFFS,FUN,OUT3,OUT4,...] = NLEVP(NAME,ARG1,ARG2,...)
%    generates the matrices and functions defining the problem specified by
%    NAME (a case insensitive string).
%    ARG1, ARG2,... are problem-specific input arguments.
%    All problems are of the form
%      T(lambda)*x = 0
%    where
%      T(lambda)= f0(lambda)*A0 + f1(lambda)*A1 + ... + fk(lambda)*Ak.
%    The matrices A0, A1, ..., Ak are returned in a cell array:
%    COEFFS = {A0,...,Ak}.
%    FUN is a function handle that can be used to evaluate the functions
%    f1(lambda),...,fk(lambda).  For a scalar lambda,
%    F = FUN(lambda) returns a row vector containing
%      F = [f1(lambda), f2(lambda), ..., fk(lambda)].
%    If lambda is a column vector, FUN(lambda) returns a row per element in
%    lambda.
%    [F,FP] = FUN(lambda) also returns the derivatives
%      FP = [f1'(lambda), f2'(lambda), ..., fk'(lambda)].
%    [F,FP,FPP,FPPP,...] = FUN(lambda) also returns higher derivatives.
%    OUT3, OUT4, ... are additional problem-specific output arguments.
%    See the list below for the available problems.
%
%  PROBLEMS = NLEVP('query','problems') or NLEVP QUERY PROBLEMS
%    returns a cell array containing the names of all problems
%    in the collection.
%  NLEVP('help','name') or NLEVP HELP NAME
%    gives additional information on problem NAME, including number and
%    meaning of input/output arguments.
%  NLEVP('query','name') or NLEVP QUERY NAME
%    returns a cell array containing the properties of the problem NAME.
%  PROPERTIES = NLEVP('query','properties') or NLEVP QUERY PROPERTIES
%    returns the properties used to classify problems in the collection.
%  NLEVP('query',property1,property2,...) or NLEVP QUERY PROPERTY1 ...
%    lists the names of all problems having all the specified properties.
%
%  [T,TP,TPP,...] = NLEVP('eval',NAME,LAMBDA,ARG1,ARG2,...)
%  evaluates the matrix function T and its derivatives TP, TPP,...
%  for problem NAME at the scalar LAMBDA.
```

```
%
% NLEVP('version') or NLEVP VERSION
%   prints version, release date, and number of problems
%   of the installed NLEVP collection.
% V = NLEVP('version')
%   returns a structure V containing version information.
%   V consists of the fields v.number, v.date, and v.problemcount.
%
% Available problems:
%
% acoustic_wave_1d   Acoustic wave problem in 1 dimension.
% acoustic_wave_2d   Acoustic wave problem in 2 dimensions.
% bcc_traffic        QEP from stability analysis of chain of non-identical cars.
%                    under bilateral cruise control.
% bent_beam          6-by-6 NEP from a bent beam model.
% bicycle            2-by-2 QEP from the Whipple bicycle model.
% bilby              5-by-5 QEP from Bilby population model.
% bucking_plate      3-by-3 NEP from a bucking plate model.
% butterfly          Quartic matrix polynomial with T-even structure.
% canyon_particle    NEP from the Schrödinger equation on a canyon-like shape.
% cd_player          QEP from model of CD player.
% circular_piston    Sparse QEP from model of circular piston.
% closed_loop        2-by-2 QEP associated with closed-loop control system.
% concrete           Sparse QEP from model of a concrete structure.
% damped_beam        QEP from simply supported beam damped in the  middle.
% damped_gyro        QEP of a damped gyroscopic system.
% deformed_consensus n-by-n QEP from multi-agent systems theory.
% dirac              QEP from Dirac operator.
% disk_brake100      100-by-100 QEP from a disk brake model.
% disk_brake4669     4669-by-4669 QEP from a disk brake model.
% distributed_delay1 3-by-3 NEP from distributed delay system.
% elastic_deform     QEP from elastic deformation of anisotropic material.
% fiber              NEP from fiber optic design.
% foundation         Sparse QEP from model of machine foundations.
% gen_hyper2         Hyperbolic QEP constructed from prescribed eigenpairs.
% gen_tantipal2      T-anti-palindromic QEP with eigenvalues on the unit
%                    circle.
% gen_tpal2          T-palindromic QEP with prescribed eigenvalues on the
%                    unit circle.
% gun                NEP from model of a radio-frequency gun cavity.
% hadeler            NEP due to Hadeler.
% hospital           QEP from model of Los Angeles Hospital building.
% intersection       10-by-10 QEP from intersection of three surfaces.
% loaded_string      REP from finite element model of a loaded vibrating
%                    string.
% metal_strip        QEP related to stability of electronic model of metal
%                    strip.
% mirror             Quartic PEP from calibration of cadioptric vision system.
% mobile_manipulator QEP from model of 2-dimensional 3-link mobile manipulator.
% nep1               2-by-2 basic NEP example.
% nep2               3-by-3 basic NEP example.
% omnicam1           9-by-9 QEP from model of omnidirectional camera.
% omnicam2           15-by-15 QEP from model of omnidirectional camera.
% orr_sommerfeld     Quartic PEP arising from Orr-Sommerfeld equation.
% pdde_stability     QEP from stability analysis of discretized PDDE.
% pdde_symmetric     n-by-n NEP from a partial delay differential equation.
% photonic_crystal   REP from dG-FEM of wave propagation in a periodic
```

```
%                      medium.
%  pillbox_cavity      170562-by-170562 NEP from a RF pillbox cavity.
%  pillbox_small       20-by-20 NEP from a RF pillbox cavity.
%  planar_waveguide    Quartic PEP from planar waveguide.
%  plasma_drift        Cubic PEP arising in Tokamak reactor design.
%  power_plant         8-by-8 QEP from simplified nuclear power plant problem.
%  qep1                3-by-3 QEP with known eigensystem.
%  qep2                3-by-3 QEP with known, nontrivial Jordan structure.
%  qep3                3-by-3 parametrized QEP with known eigensystem.
%  qep4                3-by-4 QEP with known, nontrivial Jordan structure.
%  qep5                3-by-3 nonregular QEP with known Smith form.
%  railtrack           QEP from study of vibration of rail tracks.
%  railtrack_rep       REP from study of vibration of rail tracks.
%  railtrack2          Palindromic QEP from model of rail tracks.
%  railtrack2_rep      REP from model of rail tracks.
%  relative_pose_5pt   Cubic PEP from relative pose problem in computer vision.
%  relative_pose_6pt   QEP from relative pose problem in computer vision.
%  sandwich_beam       NEP from model of a clamped sandwich beam.
%  schrodinger         QEP from Schrodinger operator.
%  shaft               QEP from model of a shaft on bearing supports with a
%                      damper.
%  sign1               QEP from rank-1 perturbation of sign operator.
%  sign2               QEP from rank-1 perturbation of 2*sin(x) + sign(x)
%                      operator.
%  sleeper             QEP modelling a railtrack resting on sleepers.
%  speaker_box         QEP from finite element model of speaker box.
%  spring              QEP from finite element model of damped mass-spring
%                      system.
%  spring_dashpot      QEP from model of spring/dashpot configuration.
%  surveillance        27-by-20 QEP from surveillance camera callibration.
%  time_delay          3-by-3 NEP from a time-delay system.
%  time_delay2         2-by-2 NEP from a time-delay system.
%  utrecht1331         QEP  1331-by-1331 QEP with singular A1.
%  wing                3-by-3 QEP from analysis of oscillations of a wing in
%                      an airstream.
%  wiresaw1            Gyroscopic system from vibration analysis of wiresaw.
%  wiresaw2            QEP from vibration analysis of wiresaw with viscous
%                      damping.
%
%  Examples:
%  coeffs = nlevp('railtrack')
%    generates the matrices defining the railtrack problem.
%  nlevp('help','railtrack')
%    prints the help text of the railtrack problem.
%  nlevp('query','railtrack')
%    prints the properties of the railtrack problem.
%
%  For example code to solve all polynomial eigenvalue problems (PEPs)
%  in this collection of dimension < 500 with MATLAB's POLYEIG
%  see NLEVP_EXAMPLE.M.

%  Reference:
%  T. Betcke, N. J. Higham, V. Mehrmann, C. Schroeder, and F. Tisseur.
%  NLEVP: A Collection of Nonlinear Eigenvalue Problems,
%  ACM Transactions on Mathematical Software, 39(2), pp7:1-7:28, 2013.
```

```
% Check inputs
if nargin < 1, error('Not enough input arguments'); end
if ~ischar(name), error('NAME must be a string'); end

name = lower(name);

if strcmp(name,'query')
    if nargin == 1
       error('Not enough input arguments')
    end
    [varargout{1:nargout}] = nlevp_query(varargin{:});
    return
end

if strcmp('string',name)
    name = 'spring';
    warning('NLEVP:string_renamed','Problem string has been renamed spring.')
end

if strcmp('version',name)
    [varargout{1:nargout}] = nlevp_version(varargin{:});
    return
end

switch name
    case 'help'
        nlevp_home = which('nlevp');
        nlevp_home = strrep(nlevp_home, 'nlevp.m', '');
        if nargin < 2
           help nlevp
        else
           if ~nlevp_isoctave
               % some nlevp are shadowed by MATLAB functions. This fixes it
               if ispc % we need to see if we are on Windows or not
               help(sprintf('%sprivate\\%s', nlevp_home, varargin{1}))
               else
               help(sprintf('%sprivate/%s', nlevp_home, varargin{1}))
               end
           else
               % Uglier code necessary for Octave.
               eval(['help ', varargin{1}]);
           end
        end
    case 'eval'
        [varargout{1:max(nargout,1)}] = nlevp_eval(varargin{:});
    otherwise
        [varargout{1:nargout}] = feval(name,varargin{:});
end

end
```

5 The MATLAB Function nlevp_example

The toolbox contains a function `nlevp_example.m` that illustrates the use of `nlevp`. Running it provides a quick test that the toolbox is correctly installed. This function can be adapted in order to test the user's own methods on subsets of NLEVP problems.

```
function nlevp_example(fname)
%NLEVP_EXAMPLE  Run POLYEIG on PEP problems from NLEVP.
% NLEVP_EXAMPLE solves all the not-too-large PEP problems in NLEVP
% by POLYEIG, sending output to the screen.
% NLEVP_EXAMPLE(fname) directs partial output to the file named fname
% (intended for generating output for NLEVP paper).

if nargin == 0
   fid = 1;
else
   fid = fopen(fname,'w');
end
s_rand = warning('off', 'NLEVP:random');  % For gen_hyper2.

nmax = 500;
probs = nlevp('query','pep');
nprobs = length(probs);
nprobs_total = length(nlevp('query','problems'));
fprintf(fid,'NLEVP contains %2.0f problems in total,\n', nprobs_total);
fprintf(fid,'of which %2.0f are polynomial eigenvalue problems (PEPs).\n', nprobs);
fprintf(fid,'Run POLYEIG on the PEP problems of dimension at most %2.0f:\n\n',nmax);

fprintf(fid,'             Problem   Dim  Max and min magnitude of eigenvalues\n');
fprintf(fid,'             -------   ---  ------------------------------------\n');
m2 = 7;
m1 = ceil(nprobs/m2);
j = 1;
for i=1:nprobs
   if fid ~= 1 && i == 9
      fprintf(fid,'                 ...\n');
      fid_save = fid;
      fid = 1;  % Omit output from this point on when writing to file.
   end
   coeffs = nlevp(probs{i});
   [n, nc] = size(coeffs{1});
   if n >= nmax
      fprintf(fid,'%20s   %3.0f is a PEP but is too large for this test.\n', ...
              probs{i}, n);
   elseif n ~= nc
      fprintf(fid,'%20s   %3.0f is a PEP but is nonsquare.\n', probs{i}, n);
   else

      % POLYEIG will convert sparse input matrices to full.
      e = polyeig(coeffs{:});
      fprintf(fid,'%20s   %3.0f  %9.2e, %9.2e\n', ...
               probs{i}, n, max(abs(e)), min(abs(e)));
      subplot(m1,m2,j)
      plot(real(e), imag(e),'.')
      title(probs{i},'Interpreter','none')
      % Tweaks.
      if strcmp(probs{i},'sign1'), ylim([-1 1]*1.5), end
      if strcmp(probs{i},'damped_beam')
         title(['         ' probs{i}],'Interpreter','none')
      end
      if strcmp(probs{i},'relative_pose_6pt')
         title(['          ' probs{i}],'Interpreter','none')
      end
```

```
      if strcmp(probs{i},'speaker_box') || strcmp(probs{i},'intersection')
         title(['     ' probs{i}],'Interpreter','none')
      end
      j = j+1;
   end
end

if nargin > 0, fclose(fid_save); end
warning(s_rand)

end
```

Part of the output of the function is shown in [1].

6 Contributing to the Collection

Contributions of suggested new problems for the collection are welcome. They can be sent to Nick Higham (`nick.higham@manchester.ac.uk`), Gian Maria Negri Porzio (`gianmaria@manchester.ac.uk`), Françoise Tisseur (`francoise.tisseur@manchester.ac.uk`). The following rules should be followed when providing new problems.

Write a LaTeX file called `problem_name.tex`, where `problem_name` is the proposed name of your example, describing the problem. Here, `problem_name` should be a string in lower case without any spaces. The `tex` file should consist of a `problem` environment, with first line stating the relevant identifiers for the problem (these properties are listed by `nlevp query properties`, and are explained in the companion document [1]):

```
\begin{problem}{problem_name}{identifier1,identifier2,...}
This is a xxx-problem of dimension nnn.
It arises in ...
\end{problem}
```

Provide your citations in a `bib` file; one `bib` file suffices even if multiple `tex` files are provided.

Write an M-file generating the coefficients of the example called `problem_name.m`. Document the M-file in the leading comment lines with the most important information from the `tex` file. If the problem is parameter dependent, set default values for any parameters not specified when the function is called. If you need extra data files, their names should begin with `problem_name`, e.g., `problem_name.mat`.

To specify a polynomial problem the first output of the M-file should be a cell array containing the coefficient matrices starting with the constant term. Thus if the first output is called `coeffs` and you want to define a PEP $P(\lambda) = \sum_{i=0}^{k} \lambda^i A_i$, then `coeffs{1}`$=A_0$, `coeffs{2}`$=A_1$, ..., `coeffs{k+1}`$=A_k$.

The second output argument must be a function that computes the nonlinear scalar functions in the definition of the problem and their derivatives; for a polynomial eigenvalue problem this is trivially provided by a line of the form

```
fun = @(lam) nlevp_monomials(lam,k);
```

Here, `nlevp_monomials.m` is a function provided with NLEVP in the `private` directory.

The third output must be a function handle that copies the behaviour of

```
>> F = @(lam) nlevp('eval',name,lam)
```

i.e., it must be the matrix function $F(\lambda) = \sum_{i=0}^{k} f_i(\lambda) A_i$; for a polynomial eigenvalue problem this is trivially provided by a line of the form

```
>> F =  nlevp_handleQEP(coeffs);
```

If one or more of the coefficients has a low-rank structure, then please the fourth output `xcoeffs` and the identifier `low-rank`. It must be a $2 \times (k+1)$ cell array such that if the i-th coefficient matrix $A_i = E_i F_i$ for some rectangular matrices E_i and F_i, then E_i is equal to `xcoeffs{1,i}` and F_i to `xcoeffs{2,i}`.

If a supposed solution is provided it should be returned in a structure `sol` with the following format:

`sol.eval`: an $m \times 1$ vector, where m eigenvalues are provided,

`sol.evec`: an $m \times n$ matrix, where column `j` is the eigenvector corresponding to `sol.eval(j)`.

If both left and right eigenvectors are known, they should be returned in `sol.evec_left` and `sol.evec_right`.

References

[1] T. Betcke, N. J. Higham, V. Mehrmann, C. Schröder, and F. Tisseur. NLEVP: A collection of nonlinear eigenvalue problems. *ACM Trans. Math. Software*, 39(2):7:1–7:28, Feb. 2013.

[2] GNU Octave. `http://www.octave.org`.

[3] N. J. Higham, G. M. N. Porzio, and F. Tisseur. An updated set of nonlinear eigenvalue problems. MIMS EPrint 2019.5, Manchester Institute for Mathematical Sciences, The University of Manchester, UK, Mar. 2019. 12 pp.